A Structural Analysis of the Default Swap Market - Part 1 (Calibration)*

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September 11, 2007

Forthcoming, Journal of Investment Management

Abstract

We analyze the default swap market with the two factor I^2 structural model, which is driven by firm value and firm leverage. This paper describes a cross-market model calibration process which results in close alignment of our model spreads with the market. This enables us to extract systematic effects reflected in the dynamics of average levels of model inputs and outputs, and discern relative value among credits by analyzing model errors. We analyze the relative value application of our model in a companion paper.

*We thank Kay Giesecke for his input at an early stage of the project; Sanjiv Das and an anonymous referee for helpful comments; Jason Kremer, David Soronow and Vishal Gupta for critical evaluation of model outputs and software engineering; Stephanie Winters for multimedia assistance; and conference organizers and attendees at the March 2006 GARP Workshop on Credit Risk Methodologies, the August 2006 Stanford Conference on Quantitative Finance, and INFORMS 2006.

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1 Introduction

Credit markets are being transformed at a phenomenal pace with transacted notional amounts in credit derivatives reaching new heights every year. Credit default swap indices have brought a level of standardization to the securities they comprise. This facilitates analysis that relates credit spreads to observable information in equity markets, which offer a transparent and liquid source of information.

The link between equity markets and credit spreads is well developed under the structural paradigm. The prototype is Merton (1974), who proposes a model of equity as a European call option on firm value, which is the only risk factor in the model. Debt in the capital structure is assumed to be a single zero coupon bond. It follows that the difference in value between riskless and corporate debt is a put option on firm value and it is straightforward to calculate the *Merton credit spread.* While Merton (1974) has been the foundation for most subsequent credit research, it is well known that Merton spreads are too low by empirical standards (see Huang and Huang (2003) and Eom et al. (2004) for empirical analysis of several structural models, including the Merton model). This difficulty is addressed partially in a first passage model, in which a firm defaults when its value first falls below a barrier. First passage models are developed in Black and Cox (1976), Leland (1994), Longstaff and Schwartz (1995) and many other articles, and they generate model credit spreads that are closer to the market than the spreads generated in Merton (1974). Another line of extensions is to assume that firms target a certain leverage along the lines of Collin-Dufresne and Goldstein (2001). This has consequences for the relative growth rates of firm value and debt. Finally, a recent set of extensions acknowledge limited investor information and model uncertainty in model variables, such as Duffie and Lando (2001), Finkelstein et al. (2002) and Giesecke and Goldberg (2004).

We make several empirical contributions to the literature, building on the I^2 credit model developed in Giesecke and Goldberg (2004). That paper develops the idea that in addition to firm value, *firm leverage*, which is the quotient of liability by value, is an additional risk factor. This two factor model has adequate breadth to address the systematic mispricing and misalignment of credit spreads. We show that properly calibrated, this model can be successfully used for credit investment applications such as relative value assessment and portfolio construction, cross market hedging, pricing of illiquid names and monitoring for suspicious credits.

Our first contribution is to develop a parametric model that is flexible enough to describe empirical phenomenon in credit markets.

Our second contribution is based on the empirical observation that credits incorporate risks differentially from one another and over time. This leads to a model that borrows some of its design from equity risk modeling. The model is estimated on cross-sectional data for groups of similar credits. Credits are grouped by region, industry, and coarse quality.[1] Model spreads are calculated using sector-based historical recovery rates. This captures basic variation in recovery rates across calibration groups. By pooling data into fundamentally determined calibration groups and updating a collection of shared *group parameters* periodically using cross-sectional data, we extract systematic effects reflected in the dynamics of average levels of model inputs and outputs. As we show in

[1]Coarse quality designations are investment grade, high yield and not rated.

Section 4.2, the model parameters exhibit both cross-sectional and temporal dependence that mirrors the market.[2] This produces a responsive model which provides a distilled view of valuation in credit markets, and therefore can be effectively used for making credit investment decisions.

Our final contribution addresses the point that during periods of distress, e.g. the auto industry crisis in 2005, the usual inputs to a structural model are insufficient to produce good model fits given the sparsity of data for particular segments. When markets are volatile, contagion and momentum tend to have a pronounced impact on spreads. We account for this in our model by introducing equity market returns for industries and companies that have experienced significant negative movements in their equity price. This novel feature of our approach allows us to maintain aggregate alignment of model and market spreads in volatile periods, and to quantify the contribution of equity returns to spread forecasts. Empirical evidence that trailing equity returns have an impact on spreads or default probabilities can be found in Collin-Dufresne et al. (2001) and Duffie et al. (2004).

There are many applications of our implied spread model to credit investment management and we explore one of them in Part II of this article. Notably, we implement and back-test several rich-cheap portfolio construction strategies based on the differences between market and model spreads. This strategy generates significant return for most of our calibration groups between January 2004 and December 2006.

The paper is organized as follows. In Section 2 we describe the I^2 model and our calibration framework. Section 3 describes the motivation for the cross-market calibration and carefully describes our methodology. This section also contains a description of the data used in this study. Section 4 describes the resulting model parameters and spreads implied by the calibration process. Concluding remarks are in Section 5.

2 The I^2 Model

The I^2 incomplete information structural model is a first passage model with an unobservable random default barrier. This feature is economically motivated — investors who are not firm insiders do not know the level of firm value that will trigger bankruptcy. Thus, default barrier uncertainty reflects an investment risk borne by investors who are not completely informed.

Default occurs when firm value falls below the unobservable default barrier $\mathcal{D}$ or equivalently, when firm value normalized by V, its value at time 0, falls below *firm leverage* $L = \mathcal{D}/V$. Thus, if M_T denotes the minimum of firm value on $[0, T]$, the probability of default on the horizon $[0, T]$ is given by

$$P\left(\frac{M_T}{V} < L\right) = \int \Psi_T(L)\, dF(L) \tag{1}$$

F is the cumulative distribution function of the effective leverage and $\Psi_T(x)$ is the first passage probability that $M_T/V \leq x$. This shows that I^2 is a weighted average or a *mixture*

[2]An analogous approach is usually not possible when estimating physical default probabilities, largely due to the rarity of defaults.

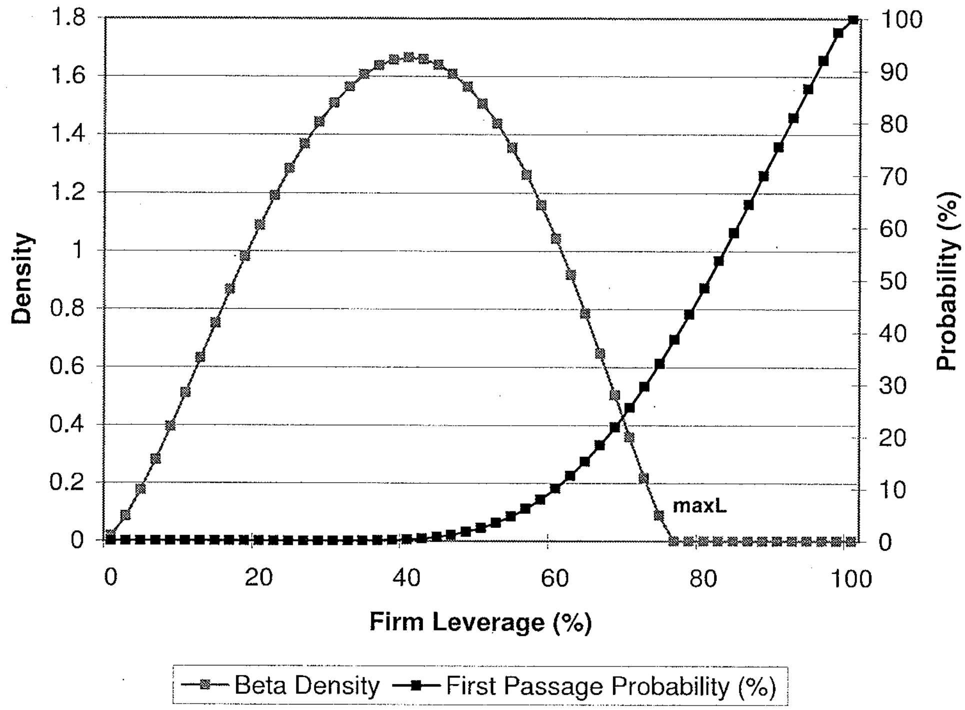

Figure 1: I^2 as a mixture.

of first passage models. Below, we specify parametric processes for the two I^2 risk factors: firm value and firm leverage.

Firm Value In the I^2 model, firm value evolves according to geometric Brownian motion whose volatility is denoted by σ. Since our analysis concerns only market prices and we do not attempt to evaluate the risk premium, we take the drift equal to the riskless interest r, which is assumed constant. Then firm value satisfies the stochastic differential equation

$$\frac{dV_t}{V_t} = rdt + \sigma dW. \tag{2}$$

It follows that V_t is lognormally distributed so that

$$V_t = Ve^{(r-\frac{\sigma^2}{2})t+\sigma W_t}. \tag{3}$$

In this situation, the first passage probability Ψ_T in formula (1) is given by

$$\Psi_T(L) = \Phi\left(\frac{L-rT}{\sigma\sqrt{T}}\right) + \exp\left(\frac{2rL}{\sigma^2}\right)\Phi\left(\frac{L+rT}{\sigma\sqrt{T}}\right)$$

where Φ denotes the normal probability distribution.

The calibration of value V and firm volatility σ_V is described in Section 3.

Firm Leverage There is no canonical choice for the distribution F of firm leverage and in the analysis below, we take F to be *scaled beta distribution.* A standard beta distribution on the interval $[0, 1]$ is specified by

$$F_{\alpha,\beta}(x) = C(\alpha, \beta) \int_0^x u^{\alpha-1}(1-u)^{\beta-1}\, du$$

where $C(\alpha, \beta)$ is a normalizing constant, see, for example, Hogg and Craig (1995, Page 180). The class of beta distributions provides a flexible array of shapes obtained by varying the parameters α and β. These parameters can be expressed in terms of expected value, μ, and variance, σ^2, of the distribution.

$$\alpha = \frac{\mu(\mu(1-\mu) - \sigma^2)}{\sigma^2} \tag{4}$$

$$\beta = \frac{(1-\mu)(\mu(1-\mu) - \sigma^2)}{\sigma^2}. \tag{5}$$

The firm leverage distribution F is obtained by rescaling $F_{\alpha,\beta}$ to the interval $[0, \max_L]$ where $\max_L$ is the largest possible value for the firm leverage L. Note that $\max_L \leq 1$ and equality occurs when the maximum possible default barrier is equal to V. The mean and variance of the scaled distribution are μ_L and σ_L^2, respectively. Hereafter, we describe the firm leverage distribution in terms of the economically meaningful parameters μ_L, σ_L^2 and $\max_L$.

The expected firm leverage μ_L is a component of distance to expected default, which is one risk factor in the model. The other is variance σ_L^2 of firm leverage, which is an indication of investor uncertainty about the true value of firm leverage. There is an economic argument for setting σ_L^2 to be function of expected firm leverage μ_L. Consider a firm whose expected leverage μ_L is relatively low so that its book liabilities are small compared to firm value. Even if the liabilities are off by a factor of two, leverage uncertainty is relatively small. At the other end of the spectrum, the value of a firm with relatively high expected leverage has likely fallen below the book value of its liabilities. Therefore, it is subject to scrutiny by analysts and investors and the uncertainty around its leverage is also small. Firms with mid-level expected leverage have the most latitude. This is illustrated in Figure 2, which illustrates the uncertainty around leverage for low, middle and high leverage firms.

This economic picture is consistent with the quadratic bound on leverage variance in terms of expected leverage that arises in the scaled beta distribution,

$$\sigma_L^2 < \mu_L \cdot (1 - \mu_L), \tag{6}$$

and we use this bound as a guide to setting the variance.

Mixture Figure 1 illustrates the ingredients to the mixture specified in formula (1). The beta density, which is calibrated on the left hand axis, has an expected leverage of 40% and standard deviation of 4.3%. The one year first passage curve, calibrated on the right hand axis, is based on a geometric Brownian motion with drift $\mu = 4.25\%$ and volatility $\sigma_V = 30\%$.

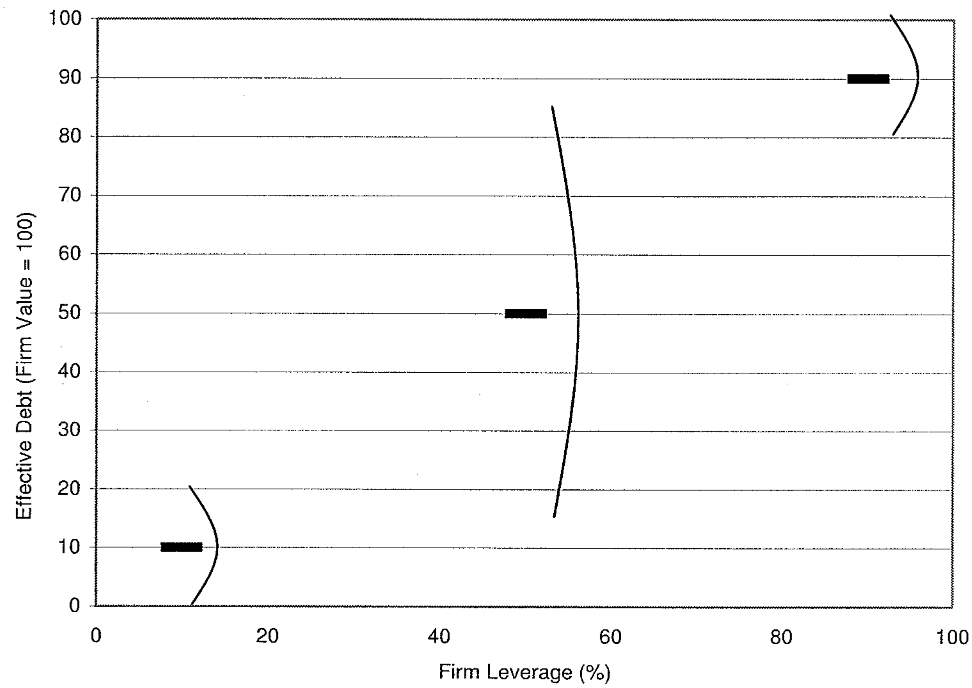

Figure 2: General form for barrier variance as a function of mean effective leverage.

Distance to Expected Default At any horizon T, the distance d_{ted} to the expected default barrier $D = \mu_L V$ is given by

$$d_{\text{ted}} = \frac{\log(V/D) + (r - \sigma_V^2/2)T}{\sigma_V \sqrt{T}} \tag{7}$$

This distribution-free quantity gives a crude but informative ranking of firms by credit-worthiness from the perspective of equity markets. In a standard first passage model, the barrier distribution $\mathcal{D}$ reduces to a point D and d_{ted} is the primary driver the probability of default. In I^2, d_{ted} determines a cohort of firms whose default probabilities are distinguished by uncertainty around firm leverage.

3 Cross Market Calibration

We simultaneously calibrate I^2 to the equity and default swap markets. The calibration uses a two step procedure called the IM (inversion-minimization) algorithm. Based on exploratory regressions we determined that region, sector and quality indicators are statistically significant. We use these indicators to aggregate firms into classes that share parameters in our calibration. This mimics a multi-factor model, where securities are pooled on the basis of their exposures to common risk drivers.

3.1 Strategy

We construct a parsimonious model $\mathcal{C}$ that expresses the default swap spread of a credit in terms of name specific variables and other market data. The model $\mathcal{C}$ consists of parameters that are shared by the members of each calibration group. For a fixed constant recovery rate R, the model spread is given by

$$S_{I^2} = s(V, \sigma_V, \mu_L, \sigma_L^2, \max_L, r; R, \mathcal{C}),$$

which is calculated by matching the value of swap leg making fixed payments at the rate of S_{I^2} to the expected value of losses due to default.

Central to the calibration is the objective function

$$\mathcal{O} = \sum \epsilon(S - S_{I^2}) \tag{8}$$

where S is the market spread and the sum is taken over all credits in a calibration group and ϵ is a *bounded influence estimator* that reduces the weight of outliers relative to a least squares minimization. We discuss bounded influence estimation and give a precise definition of the function ϵ in Section 3.4.

For each calibration group, do the following:

1. Combine observable data with an initial guess of the model parameters in $\mathcal{C}$ to set firm leverage variables μ_L, σ_L^2 and $\max_L$.
2. Repeat until convergence:
   (a) Inversion step: updates firm value variables V and σ_V for each credit using formulas (9) and (10).
   (b) Minimization step: updates firm leverage variables μ_L, σ_L^2 and $\max_L$ by modifying the model parameters $\mathcal{C}$ so as to reduce the objective (8).
3. Calculate model spreads S_{I^2}.

Remark 3.1. *Note that the variance of firm value is updated in both the inversion and minimization steps. This is done in order to match market and model spread term structures.*

3.2 Inversion Step: Updating Firm Value Variables

We calibrate firm value and volatility under the assumption that equity is a perpetual barrier call option on firm value with a fixed default barrier D[3], so that

$$E = V - D\left(\frac{D}{V}\right)^{\gamma} \tag{9}$$

[3]We were not successful in implementing the full model, with equity as a perpetual barrier call option on firm value with a random default barrier, consistently across all credits. In cases where the calibration was successful the results were only marginally different.

where $\gamma = 2r/\sigma_V^2$. A derivation can be found in Merton (1974) or Goldberg et al. (2006). From Ito's Lemma, we obtain

$$E\sigma_E = \frac{\partial E}{\partial V} V \tilde{\sigma}_V. \tag{10}$$

Values for V and $\tilde{\sigma}_V$ are obtained by inverting formulas (9) and (10). The inverse problem is not well posed. However, Goldberg et al. (2006) give necessary and sufficient conditions that guarantee a unique solution. A convenient formulation is in terms of the equity normalized default barrier

$$d = \frac{D}{E}.$$

Proposition 3.2. *Given values of equity $E > 0$, equity volatility $\sigma_E > 0$, and the riskless rate $r > 0$, there is a limit $d_{\max} = d_{\max}(\sigma_E, r) > 0$ that makes the inverse problem well posed: There are unique values for initial firm value V and firm volatility σ_V if and only if*

$$d \leq d_{\max}.$$

This step is performed at every iteration for each credit in a calibration group. The parameters of the model are held constant during the inversion step. Next, we discuss how the firm leverage variables are updated.

3.3 Minimization Step: Updating Firm Leverage Variables and Firm Volatility

The parameters of the firm leverage distribution are set in terms of a combination of idiosyncratic information and the model parameters in $\mathcal{C}$.

3.3.1 Updating expected firm leverage with the parameter c_{mean}.

Parameter c_{mean} is central in determining the empirical adjustments that are necessary to translate debt on balance sheet for each credit into the expected value of their default barrier which is used in calculating model spreads. This parameter provides feedback between the inversion and minimization steps as the model finds a balance between adjusting model leverage and changing other parameters in order to achieve the best fit possible to credit spreads.

In a preliminary step, we set the level of liabilities individually for each firm from balance sheet and off balance sheet data:

$$D_{\text{init}} = 0.5D_S + 0.5D_U + 0.6D_L \tag{11}$$

where D_S is short term debt, D_U is unsecured long term debt and D_L is secured long term debt. Our definition of long term debt includes operating leases and pension plan shortfalls. This initial estimate D_{init} of the default barrier D is an important ingredient

to the relative ordering by expected leverage $L = D/V$ among credits in each calibration group.

Set $d_{\text{init}} = D_{\text{init}}/E$. This initial estimate of the equity normalized default barrier d is lacking in two ways. First, it may exceed the limit imposed by Proposition 3.2. If $d_{\text{init}} > d_{\text{max}}$, it is impossible to find compatible values of V and σ_V. Second, even if $d_{\text{init}} \leq d_{\text{max}}$ so the inverse problem is well posed, the initial estimate may not reflect the relative importance of leverage as a determinant of credit spreads. This importance varies over time and by calibration group.

Therefore, in a second step, we adjust the spacing between the values of d within each calibration group using the parameter c_{mean}. This parameter adjusts the initial level of equity normalized liabilities d_{init} for all credits in the calibration group simultaneously.

Set equity normalized liabilities to

$$d = d_{\text{init}}(1 - N(\delta_1)) + d_{\text{max}}N(\delta_2) \tag{12}$$

$$= d_{\text{max}} - (d_{\text{max}}N(-\delta_2) - d_{\text{init}}N(-\delta_1)) \tag{13}$$

where

$$\delta_1 = \frac{\log(d_{\text{init}}/d_{\text{max}}) + 0.5c_{\text{mean}}^2}{c_{\text{mean}}} \tag{14}$$

$$\delta_2 = \delta_1 - c_{\text{mean}}. \tag{15}$$

Formula (13) is borrowed from the theory of option pricing for its shape. The parameter c_{mean} plays the role of volatility in the value of a put option that is subtracted from the maximum level of equity normalized liabilities d_{max}. Figure 3 shows that an increase in c_{mean} tends to cluster the equity normalized liabilities while preserving their order. This increase affects leverage $L = D/V$ in approximately the same way.

Given an estimate of equity normalized liabilities d, we set the expected value of the firm leverage distribution as

$$\mu_L = \frac{dE}{V}.$$

The level of the parameter c_{mean} is inversely related to the importance of distance to expected default in determining market spreads. As we illustrate in Section 4, this importance varies over time and by calibration group.

3.3.2 Updating maximum firm leverage $\max_L$ with the parameter c_{span}.

The model parameter c_{span} sets the upper support of the firm leverage distribution as follows:

$$\max_L = \frac{1}{1 + c_{\text{span}}(1 - \mu_L)}.$$

We restrict the parameter $c_{\text{span}} \in [.5, 1]$ so that

$$\max_L \in [1/(1.5 - .5\mu_L), 1/(2 - \mu_L)].$$

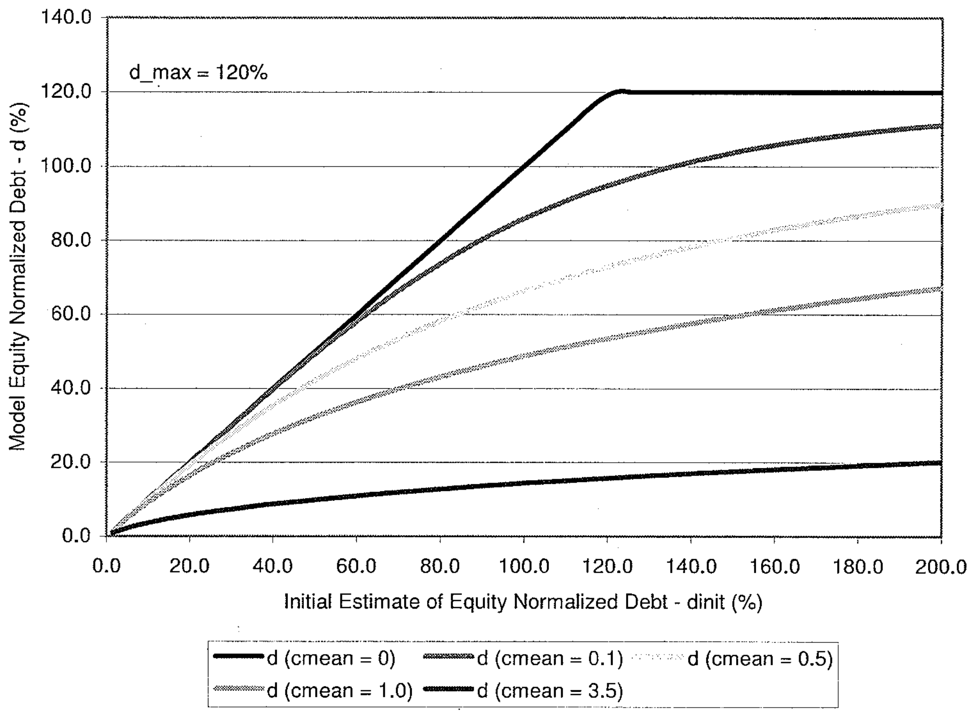

Figure 3: Effect of parameter c_{mean} on model debt.

The lower bound $c_{\text{span}} = .5$ bounds $\max_L$ away from 1. This leads to upward sloping model term structures that are compatible empirical observation for most cases. The upper bound $c_{\text{span}} = 1$ bounds $\max_L$ away from μ_L. This prevents numerical instability caused by a nearly degenerate firm leverage distribution. Figure 4 shows the relationship between the variables μ_L and $\max_L$ for different values of the parameter c_{span}.

3.3.3 Updating the variance of firm leverage with parameters $c_{\text{var}}, c_{\text{var1}} - c_{\text{var5}}$

The quadratic bound

$$\sigma_L^2 \leq \mu_L(1 - \mu_L) \tag{16}$$

inherent in the scaled beta distribution is a guide to setting the variance σ_L^2 for credits in a calibration group. However, we need to add flexibility in order to fit the the empirical distribution of default swap spreads in the group.

Firm leverage variance is given by

$$\sigma_L^2 = c_{\text{var}} v_1 v_2 v_3 \mu_L (1 - \mu_L)$$

where $c_{\text{var}} \in (0, 0.6]$ sets the overall level of firm volatility as a function of the bound and the adjustments v_i depend on model parameters, the firm specific variables μ_L and d_{ted}, and term T. These adjustments are motivated and precisely defined in Appendix A.

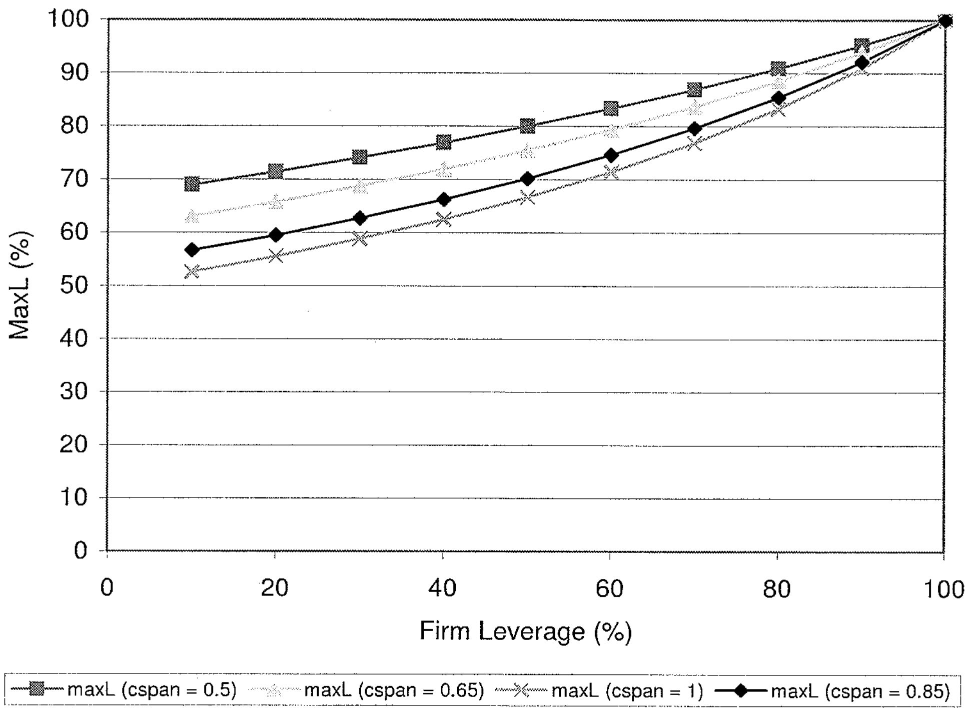

Figure 4: Effect of parameter c_{span} on the effective leverage variable $\max_L$

3.3.4 Updating the volatility of firm value with parameters $c_{\text{vol1}} - c_{\text{vol3}}$.

During periods of high volatility, certain industries or quality cohorts are affected by credit events to a larger extent than the broad sector to which they belong. We introduce *contagion* and *momentum* indices to track this phenomenon and we use these indices to make appropriate adjustments to firm volatility. An additional adjustment that depends on term and expected firm leverage is used to refine the shape of the term structure. The value of firm volatility used in the calculation of I^2 model spreads is:

$$\sigma_V = w_1 w_2 w_3 \tilde{\sigma}_V$$

where $\tilde{\sigma}_V$ is the value of firm volatility obtained from the inversion step.

The w_is, like the v_is, are motivated and precisely defined below in Appendix A.

3.4 Bounded Influence Estimation (BIE)

The calibration minimizes the objective function

$$\mathcal{O} = \sum \epsilon(S - S_{I^2}) \tag{17}$$

where S is the market spread. Since relative large errors are more prevalent than predicted by a normal distribution, we use the Huber bounded influence error estimator where the

function ϵ is defined by

$$\epsilon(x) = \begin{cases} x^2/2 & |x| \leq k \\ k(|x| - k/2) & |x| > k \end{cases} \tag{18}$$

A standard setting is $k = 1.345s$ where s is an estimate of the standard deviation of the errors. If the errors were normally distributed, this choice of k would result an estimator with 95% asymptotic efficiency. Further details are in Huber (1981). We set s to 30 basis points for investment grade credits and 100 basis points for high yield credits.

3.5 Minimization Algorithm

We use the Levenberg-Marquardt algorithm, which is a hybrid between steepest descent and Newton's method. Further details are in Press et al. (1992, Pages 683–688). Spread values from the most recent successful calibration are used as seed values.

3.6 Extrapolation

Our classification scheme, which is based on region, sector and quality extends beyond the estimation universe. We exploit this to generate model spreads for credits outside the estimation universe by using the corresponding parameter set for the group. For unrated names in the US and high yield names in EU/UK, there are not enough default swap spread data, and we use corresponding high yield and unrated parameter sets, respectively, to produce spreads in this case.

3.7 Data

The default swap spread quotes used in the calibration are provided by Credit Market Analysis (CMA) based in London. They are indicative end-of-day quotes (or firm quotes when available) collected from a consortium of brokers and dealers. Average historical recovery rates are compiled from post-default price data from a database of defaulted bonds provided by Moody's. The study period extends from January 2004 to December 2006. We calibrate the model on the earliest business day on or after the 5th, 15th and 25th day of each month. Table 1 shows summary statistics for June 15, 2006.

Table 1: Summary Statistics for the default swap universe for June 15, 2006

Region	Quality	Number	Avg. Market Spread (bps)
US	Investment Grade	321	47
	High Yield	112	246
EU/UK	Investment Grade	125	45
	Unrated	65	71

Data with large discontinuities or spikes over the past 20 trading days are dropped from the estimation universe. Any credit whose spread to firm leverage ratio exceeds 2000 is dropped and firms outside the financial sector whose spread to firm leverage ratio is less

than 50 is also dropped. To avoid spurious volatility, we include in the current estimation universe only credits for which data have been available for the two previous calibration dates.

The data are sorted into 16 calibration groups for estimation and four additional groups for extrapolation on the basis of domicile, sector classification and coarse quality. We aggregate data by sector leading to calibration groups such as (US, Investment Grade, Consumer Discretionary). Sparsely populated groups are further aggregated. A complete list of our calibration groups is in Table 2. On June 15, 2006 the group sizes in the U.S. range from US-GROUP02-IG with 17 credits to US-GROUP02-HY with 55 credits. The average calibration group has 36 credits.

Table 2: Calibration Groups

Calibration Group	Description
US-CONSDISC-IG	Consumer Discretionary
US-ENERGY-IG	Energy
US-FINANCE-IG	Finance
US-INSURANC-IG	Insurance
US-GROUP01-IG	Con. Staples, Health
US-GROUP02-IG	Info. Tech., Telecom
US-INDUST-IG	Industrials
US-MATERIAL-IG	Basic Materials
US-UTILITY-IG	Utilities
US-GROUP01-HY	Cons. Disc., Cons. Staples, Health
US-GROUP01-NR	Cons. Disc., Cons. Staples, Health
US-GROUP02-HY	Energy, Ind., Util., Matl., Info. Tech., Telecom, Finance
US-GROUP02-NR	Energy, Ind., Util., Matl., Info. Tech., Telecom, Finance
EU-FINANCE-IG	Finance
EU-GROUP01-IG	Cons. Disc., Cons. Staples, Health
EU-GROUP01-NR	Cons. Disc., Cons. Staples, Health
EU-GROUP01-HY	Cons. Disc., Cons. Staples, Health
EU-GROUP02-IG	Energy, Ind., Util., Matl., Info. Tech., Telecom
EU-GROUP02-NR	Energy, Ind., Util., Matl., Info. Tech., Telecom, Finance
EU-GROUP02-HY	Energy, Ind., Util., Matl., Info. Tech., Telecom, Finance

4 Market and I^2 implied default swap spreads

4.1 Aggregate Analysis

An alignment of model to market spreads at an aggregate level allows us to assess relative value at any point in time. Below, we examine the degree to which I^2 model spreads track the market.

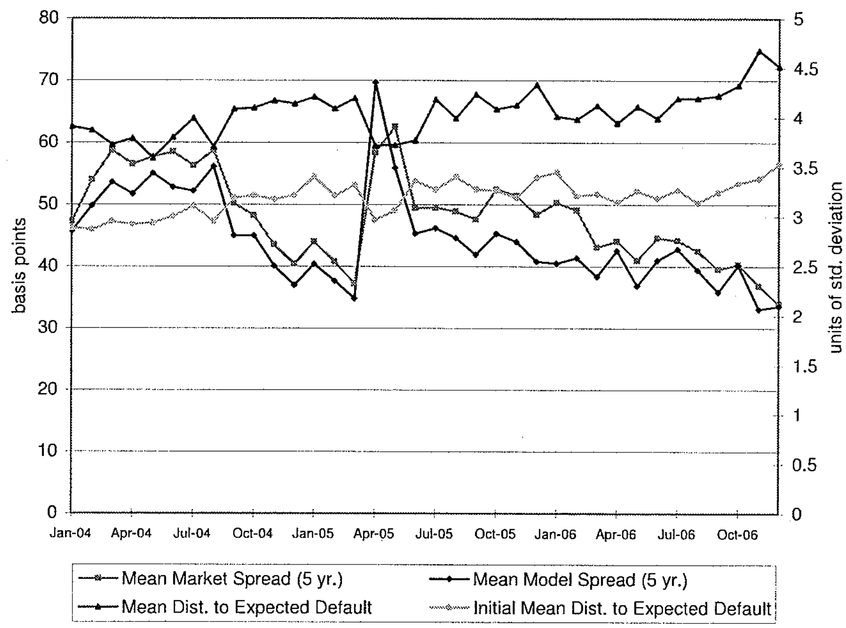

Figure 5: Mean Market and Model Spread in basis points (left axis), Mean Initial and Implied Distance to Expected Default in units of horizon standard deviation of log firm value (right axis) for US-Investment Grade.

Figures 5–8 show market and model spreads averaged over investment quality categories by region. Spreads are marked on the left axis. Notice that the average model spread tracks the average market spread successfully in all categories.

On the same graphs, we show average initial and implied distance to expected default (d_{ted}), marked on the right axis. The initial measure of d_{ted} is estimated with formula (7) using balance sheet and equity market data. The implied measure is calculated after adjusting balance sheet debt as described in section 3.3.1. A primary mechanism by which the model matches market spreads is by raising the *initial* d_{ted}, which lowers spreads. This is done using the c_{mean} parameter.

The dynamics of average implied d_{ted}, however, do not completely mirror spread movements. Some portion of the burden of reproducing spread dynamics falls on other model parameters. The spring 2005 crisis in the automotive industry is an example of spread dynamics that cannot be captured by a simple Merton-type approach. In this case, the implied d_{ted} accounts only for a small part of the spike in spreads. The remaining gap is filled by the contagion and momentum parameters.

4.2 Contribution of Second Factor

The variance of firm leverage contributes a premium over the spread level determined by the first passage model for a given distance to default. Figure 9 shows cross-sectional plots of model spreads for four representative calibration groups. The first passage model spread using a recovery rate of 40 percent is also shown for comparison. An important

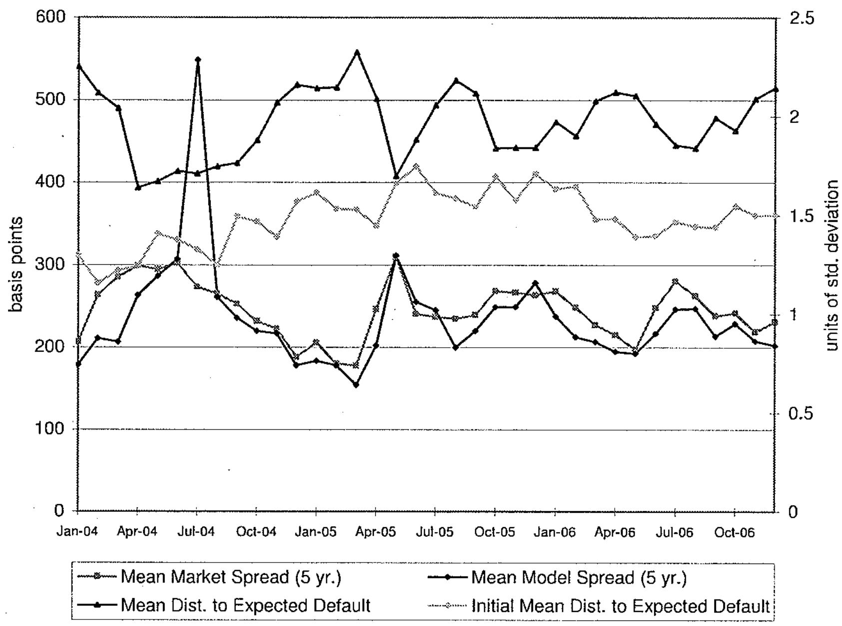

Figure 6: Mean Market and Model Spread in basis points (left axis), Mean Initial and Implied Distance to Expected Default in units of horizon standard deviation of log firm value (right axis) for US-High Yield.

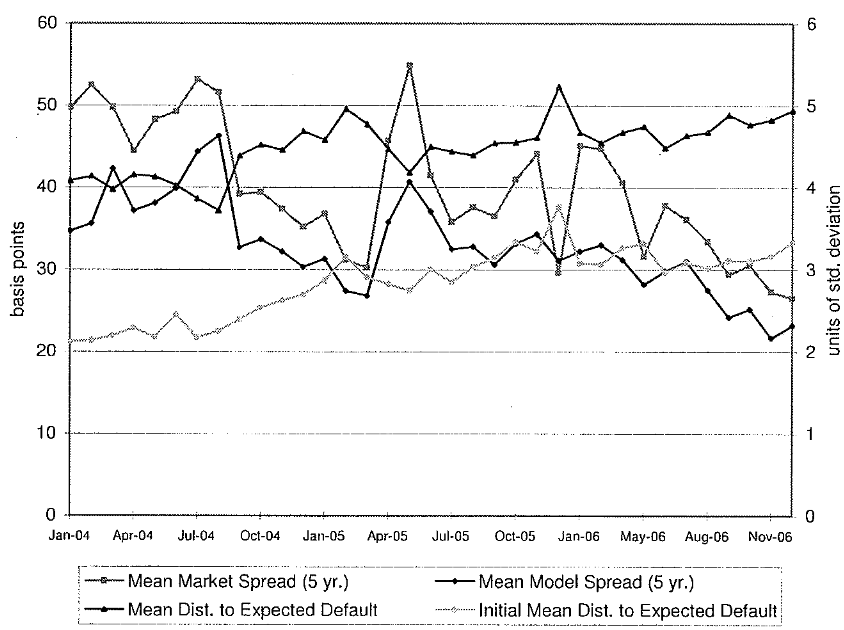

Figure 7: Mean Market and Model Spread in basis points (left axis), Mean Initial and Implied Distance to Expected Default in units of horizon standard deviation of log firm value (right axis) for EU-Investment Grade.

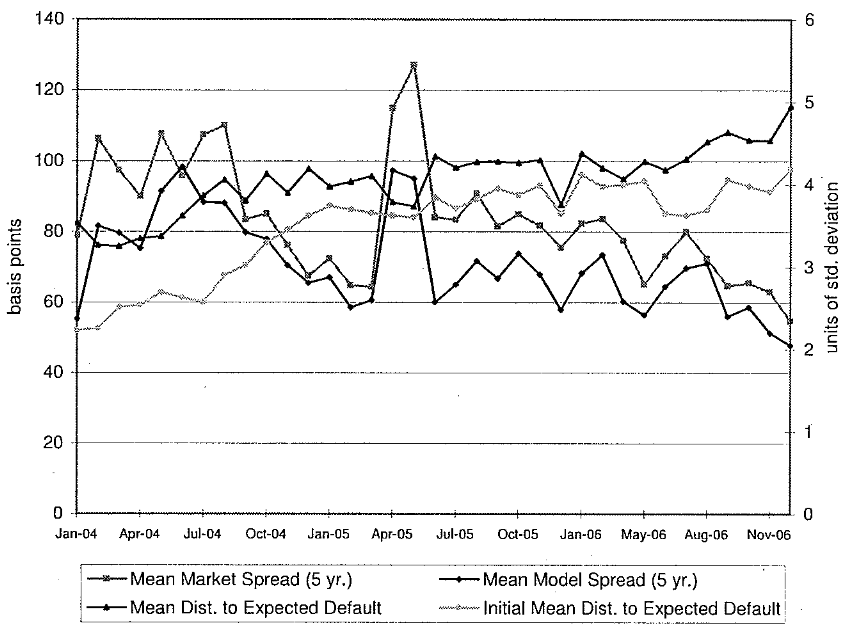

Figure 8: Mean Market and Model Spread in basis points (left axis), Mean Initial and Implied Distance to Expected Default in units of horizon standard deviation of log firm value (right axis) for EU-Unrated.

parameter determining the level of the premium is c_{var4} described in Appendix . Larger values of c_{var4} increase spreads for credits with large values of d_{ted} relative to those with small values of d_{ted} (see Figure). Figure 10 shows histories of this parameter for the four calibration groups. Notice that there are periods in the history when the parameter levels for different groups are quite separated. While changes in c_{mean} imply movement of credits along the distance to default axis, and thus along a particular model spread curve, changes in c_{var4} moves the entire model spread curve, reflecting changes in the amount of spread at a given distance to expected default. For the high yield group this parameter does not contribute much due to the fact that most of the credits in this group have relatively low values of d_{ted}.

4.3 Cross-sectional Analysis

Table 3 shows size and goodness of fit metrics for the four calibration groups for June 15, 2006. Figures 11–13 shows the distribution of market and model spreads for these groups as a function of implied d_{ted}. For investment grade groups, credits with errors larger than 30 basis points are flagged, while credits with errors larger than 100 basis points are flagged for high yield groups. These figures show how our two-factor approach captures the variation in market spreads in a nonlinear setting. The dispersion in model spreads for a particular d_{ted} level arises from differences in leverage as well as contagion and momentum effects. We test the performance of investment strategies that exploit the size and direction of the calibration errors in the next section.

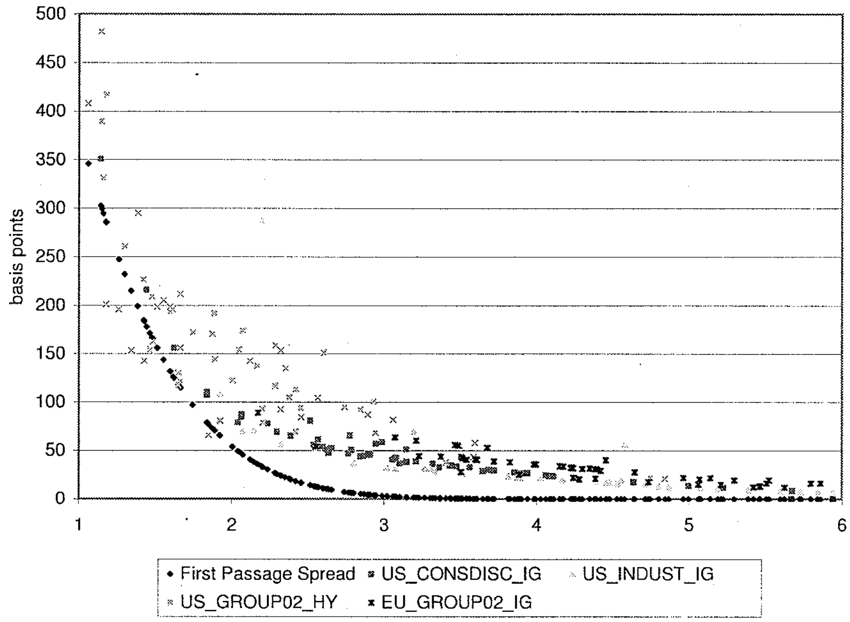

Figure 9: I^2 Model spreads for four groups and First Passage model spread

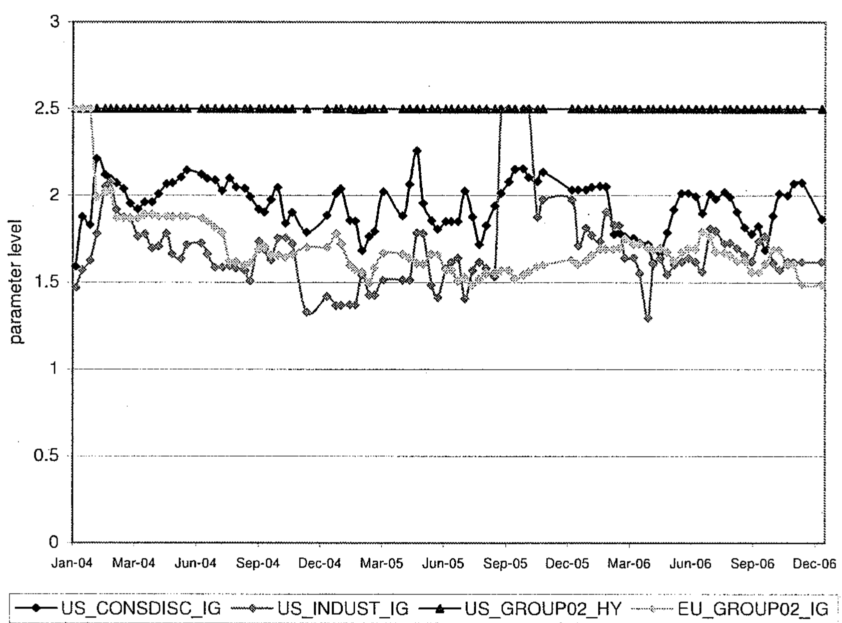

Figure 10: Histories of parameter c_{var4} for four calibration groups. Parameter can vary between 0 and 2.5

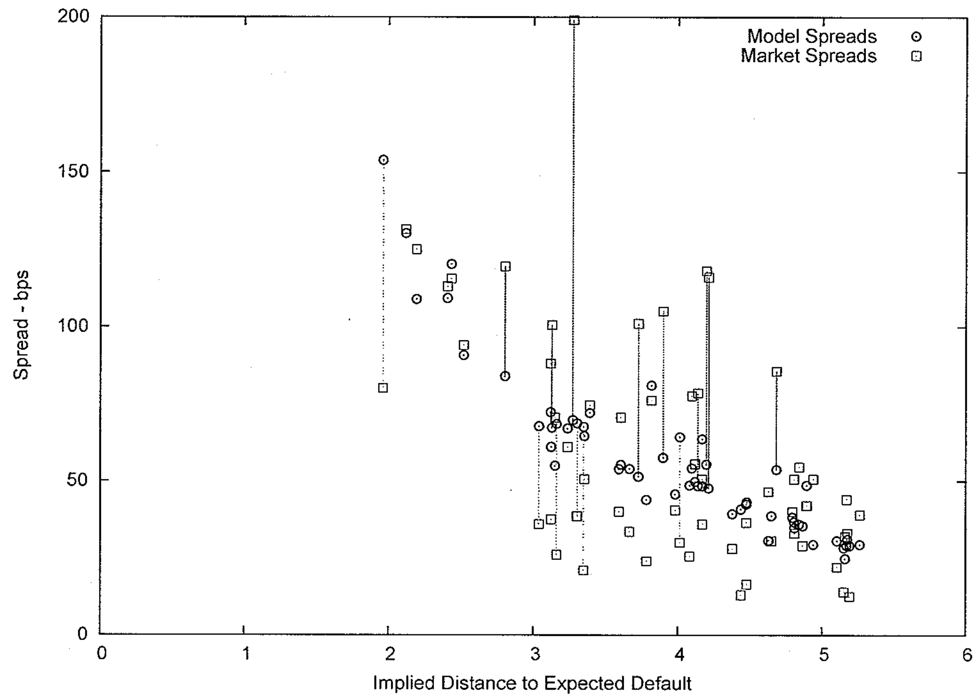

Figure 11: US - IG - Consumer Discretionary (R-Square (BIE) 0.77)

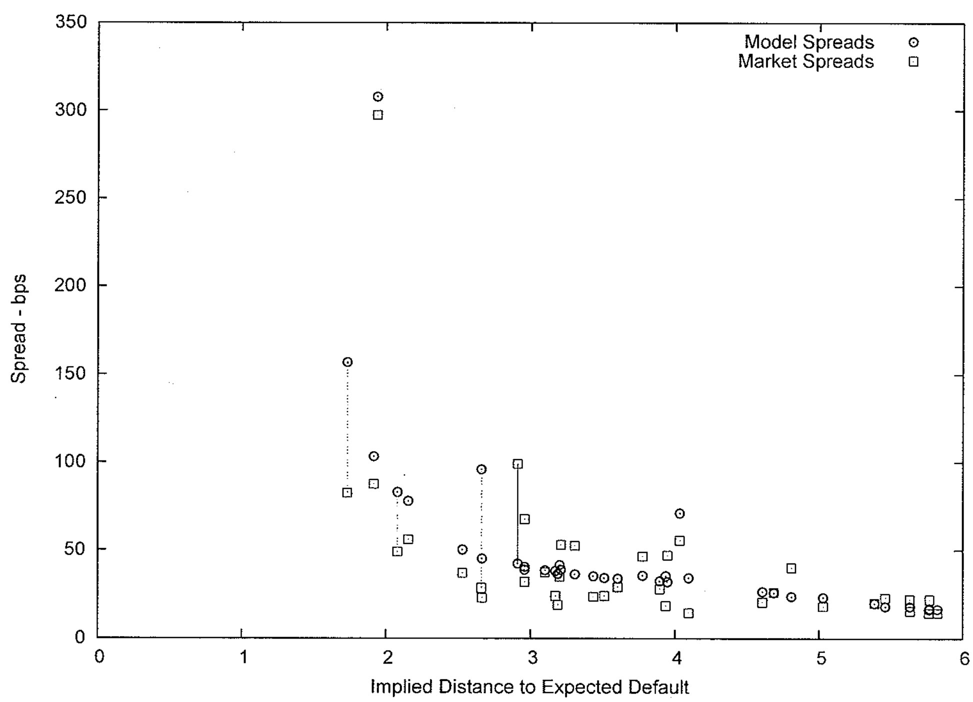

Figure 12: US - IG - Industrials - (R-Square (BIE) 0.91)

Table 3: Results for June 15 2006

Group	Industries	Names	R-Square (BIE)
US-IG-Industrials	13	35	0.85
US-IG-Consumer Discretionary	12	53	0.76
US-HY- Group 2 (Energy, Industrials, Materials) (Utility, Infotech, Telecom)	38	59	0.76
EU/UK-IG-Group 2 (Energy, Industrials, Materials) (Utility, Infotech, Telecom)	38	44	0.58

Goodness of Fit Figures 14 shows time series of goodness of fit using bounded influence errors based estimates for the four groups. For the US groups, the model is able to explain 70 to 90 percent of the total variation in CDS spreads.[4] The corresponding range for the European group is 50 to 70 percent. Apart from a period in late 2005 the levels of fit achieved are fairly consistent over time. Even during the period of the Auto Industry crisis in the summer of 2005 the fit for the US-CONSDISC-IG group which contains this industry was similar to that achieved in other periods. The lower R-Square values for the European group are due to grouping of data to super-sector and regional levels. We find, however, that this does not compromise the effectiveness of the model for applications.

4.4 Case Histories

Figures 15–18 show market and model spreads for four representative credits in the calibration groups discussed above. The figures also show the contribution (in basis points) of the contagion and momentum risk indices to the model spread. In cases where the model spread is systematically higher or lower than the market spread for a period of three months, we report a 3-month average model residual as the 'Systematic Bias' curve.

In most calibration groups, there is a core group of credits that the model tracks closely, e.g. Fedex in Figure 15. Radioshack, shown in Figure 16, is an example where there was idiosyncratic deterioration in credit quality reflected in the momentum effect contributions seen in the later part of its history. The third example, Nextel in Figure 16, shows a case where it becomes difficult to track a credit following a merger announcement. Sprint and Nextel announced a merger in Dec. 2004.[5]

5 Concluding Remarks

Using spread data to calibrate a structural model aligns model forecasts more closely with market data while retaining the value added by using equity market data in spread

[4]As the model is fit in a nonlinear setting, R-Square values are calculated using total variation of CDS spreads rather than variation around the mean.

[5]The model also often misses the increase in spread in a credit which becomes a leveraged buyout target. This exceptional situation falls outside the scope of the current implementation of the model.

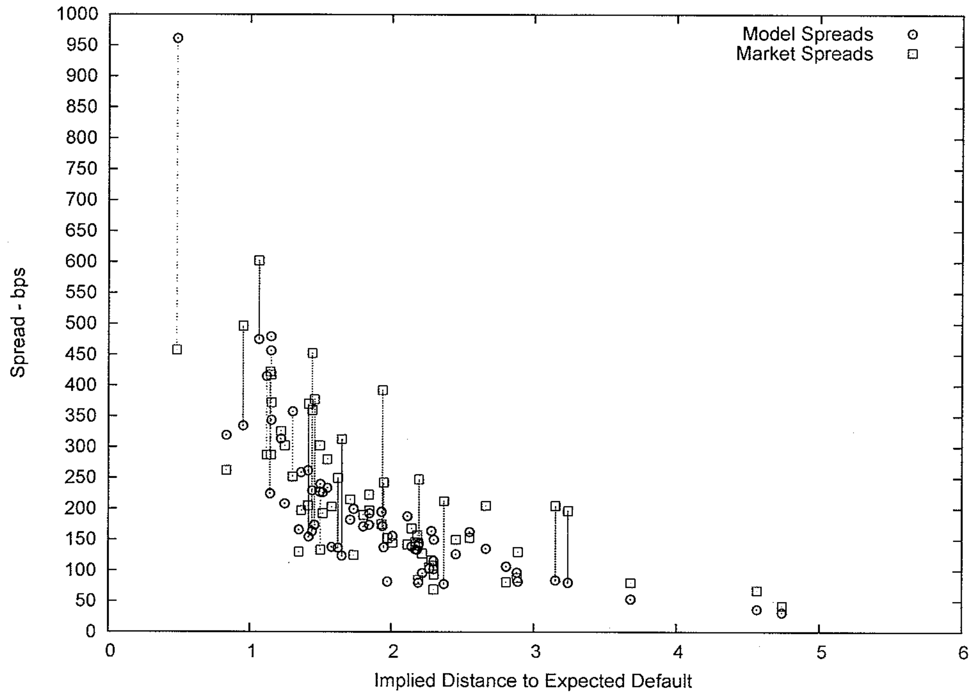

Figure 13: US - HY - Group 2 - (R-Square (BIE) 0.83)

forecasting. We generate model spread forecasts by performing a cross-sectional calibration using a next generation structural model, the I^2 model described in Goldberg and Giesecke (2004). This model, which falls among incomplete information structural models, can bring the levels of default probabilities to those consistent with pricing. For better forecasts during volatile periods, we use risk indices constructed from equity return data to better differentiate industries and credits with recent downturns in equity market. We generate model spreads for pools of credits grouped by region, sector and quality. We extend our model forecast from our calibration universe to larger set of credits by using calibrated parameters from the corresponding group. Applications of our model, including assessment of relative value, pricing of illiquid names, cross market hedging and monitoring credit portfolios are analyzed in a companion paper.

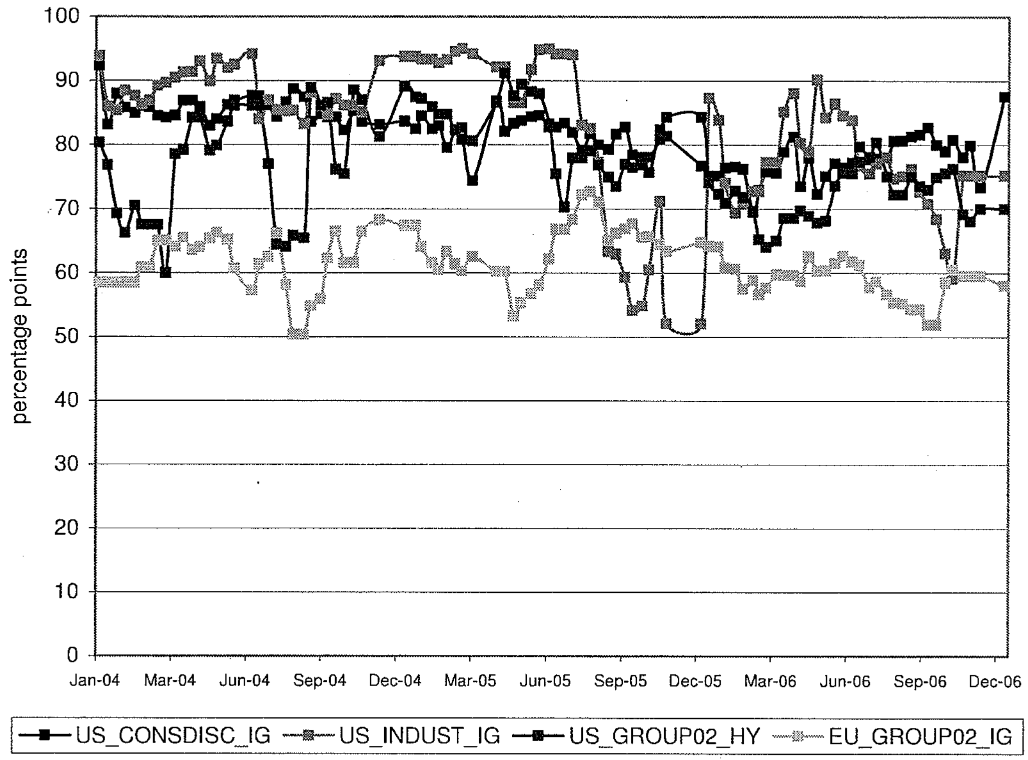

Figure 14: Goodness of fit (R-Square in percentage points) - four representative groups

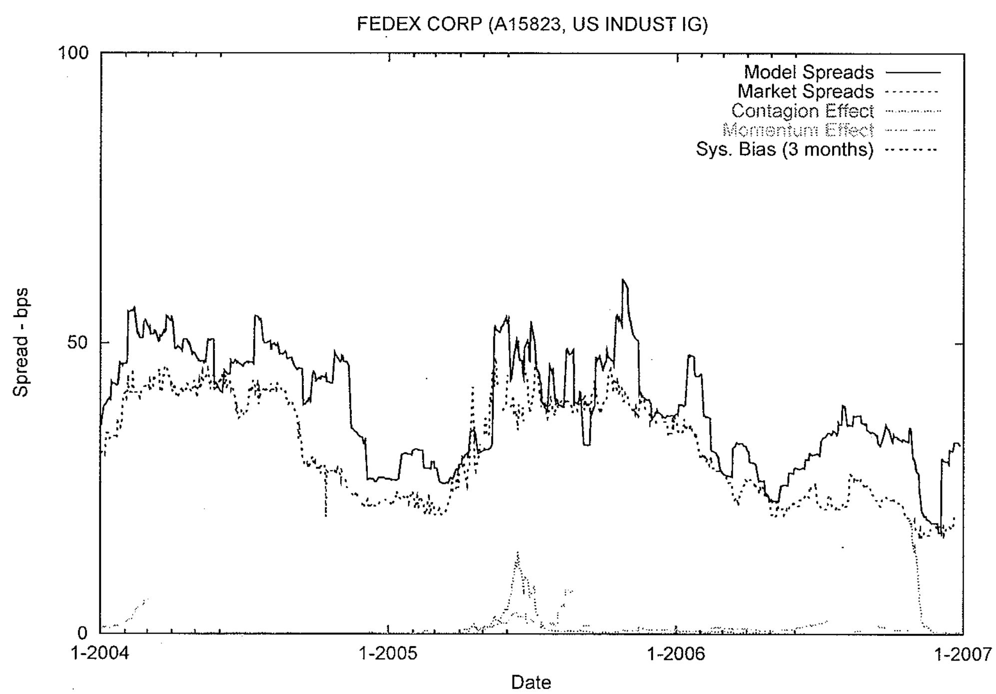

Figure 15: Market and Model Spreads for Fedex.

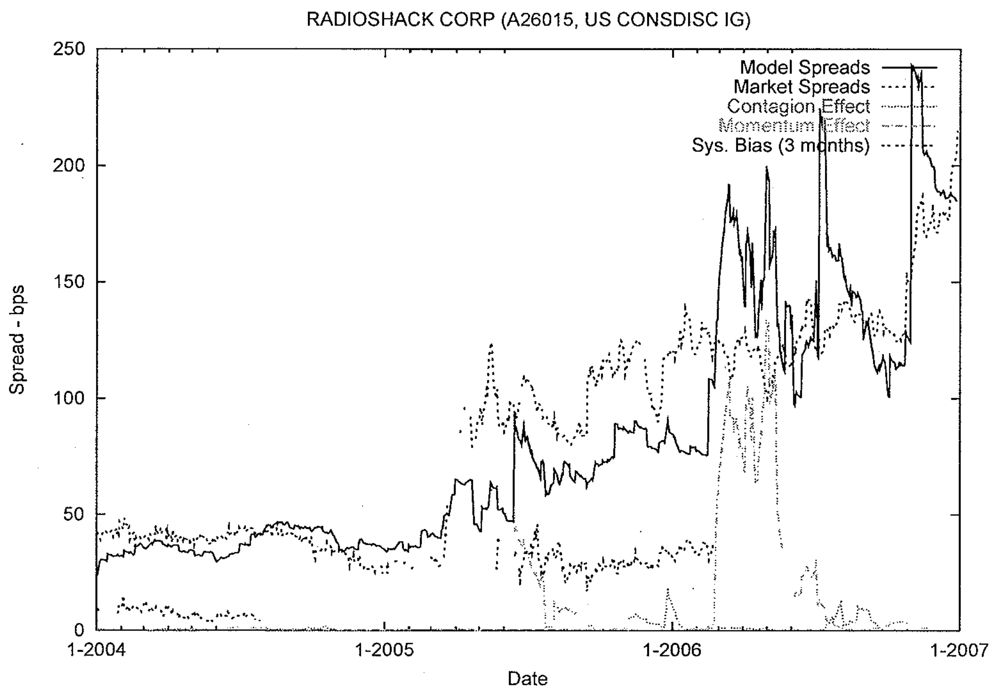

Figure 16: Market and Model Spreads for Radioshack.

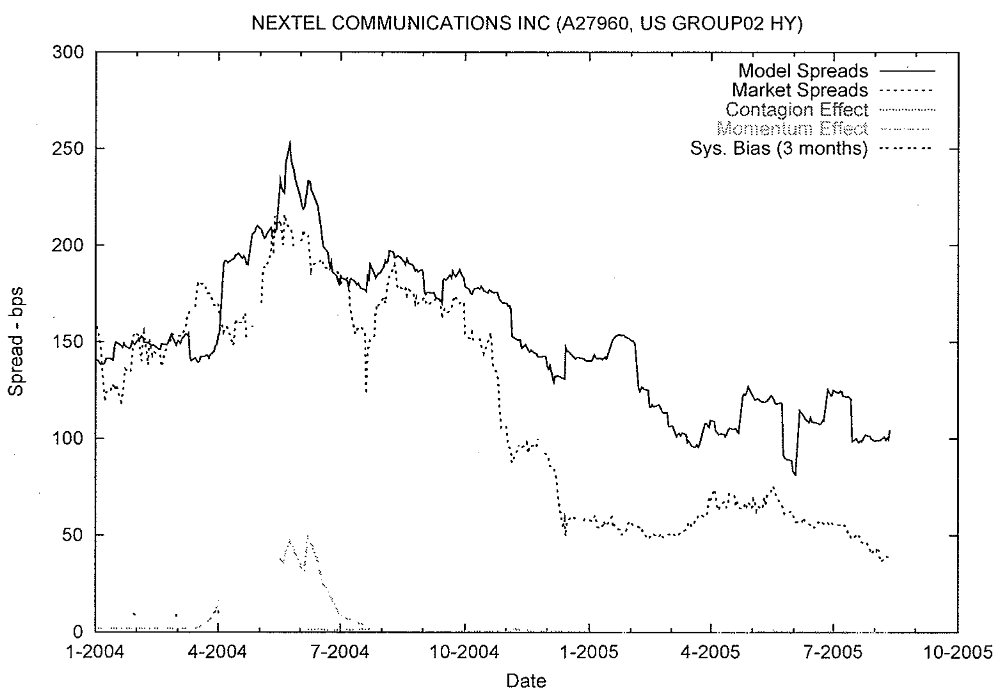

Figure 17: Market and Model Spreads Nextel.

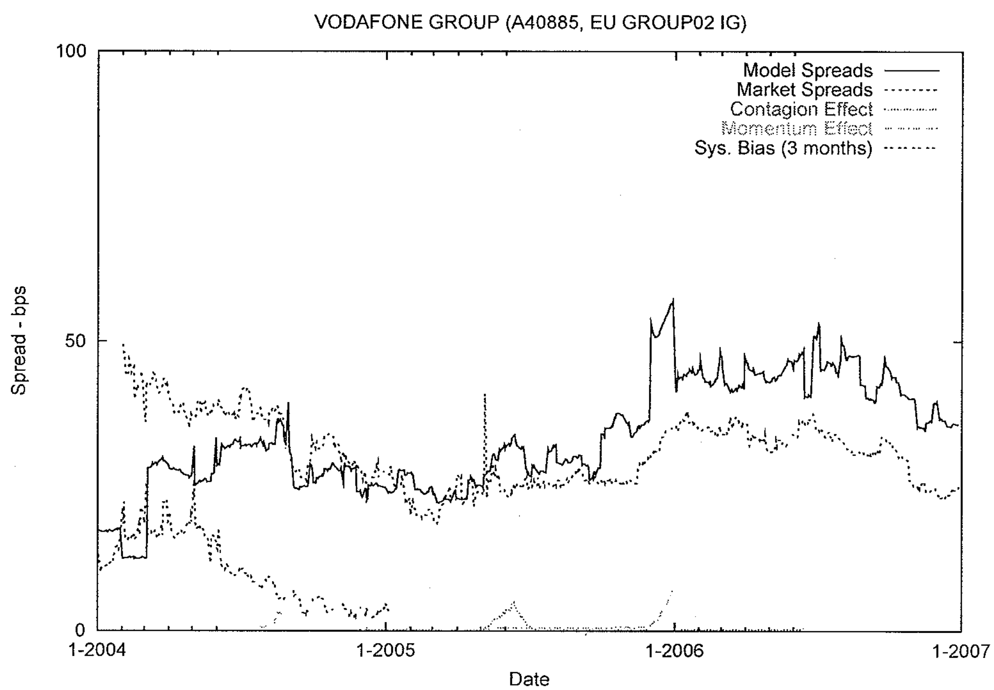

Figure 18: Market and Model Spreads for Vodafone.

References

Fischer Black and John C. Cox. Valuing Corporate Securities: Some Effects of Bond Indenture Provisions. *Journal of Finance*, 31:351–367, 1976.

Pierre Collin-Dufresne and Robert Goldstein. Do Credit Spreads Reflect Stationary Leverage Ratios? 56:1929–1958, 2001. Journal of Finance.

Pierre Collin-Dufresne, Robert Goldstein and Spencer Martin. The Determinants of Credit Spread Changes. *Journal of Finance*, 56:2177–2207, 2001.

Darrell Duffie and David Lando. Term Structures of Credit Spreads with Incomplete Accounting Information. *Econometrica*, 69(3):633–664, 2001.

Darrell Duffie, Leandro Saita and Ke Wang. Multi-Period Corporate Failure Prediction with Stochastic Covariates, 2004. To appear in Journal of Financial Economics.

Young Ho Eom, Jean Helwege and Jingzhi Huang. Structural Models of Corporate Bond Pricing: An Empirical Analysis. *Review of Financial Studies*, 17:499–544, 2004.

Vladimir Finkelstein, Jean-Pierre Lardy, George Pan and J. Tierney. CreditGrades Technical Document, 2002. Risk Metrics Group.

Kay Giesecke and Lisa Goldberg. Forecasting Default in the Face of Uncertainty. *Journal of Derivatives*, 12(1):1–15, 2004.

Lisa Goldberg, Rajnish Kamat and Vijay Poduri. First Passage Models and Inverse Problems, 2006. Working Paper, MSCI Barra.

Robert V. Hogg and Allen T. Craig. *Introduction to Mathematical Statistics.* Prentice Hall, Upper Saddle River, New Jersey, fifth edition, 1995.

Jingzhi Huang and Ming Huang. How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?, 2003. Working paper, Penn. State and Stanford University.

P.J. Huber. *Robust Statistics.* John Wiley and Sons, New York, 1981.

Hayne E. Leland. Corporate Debt Value, Bond Covenants, and Optimal Capital Structure. *Journal of Finance*, 49(4):1213–1252, 1994.

Francis A. Longstaff and Eduardo S. Schwartz. A Simple Approach to Valuing Risky Fixed and Floating Rate Debt. *Journal of Finance*, 50(3):789–819, 1995.

Robert C. Merton. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance*, 29:449–470, 1974.

William H. Press, William T. Vetterling, Saul A. Teukolsky and Brian P. Flannery. *Numerical Recipes in C.* Cambridge University PRess, Cambridge, 1992.

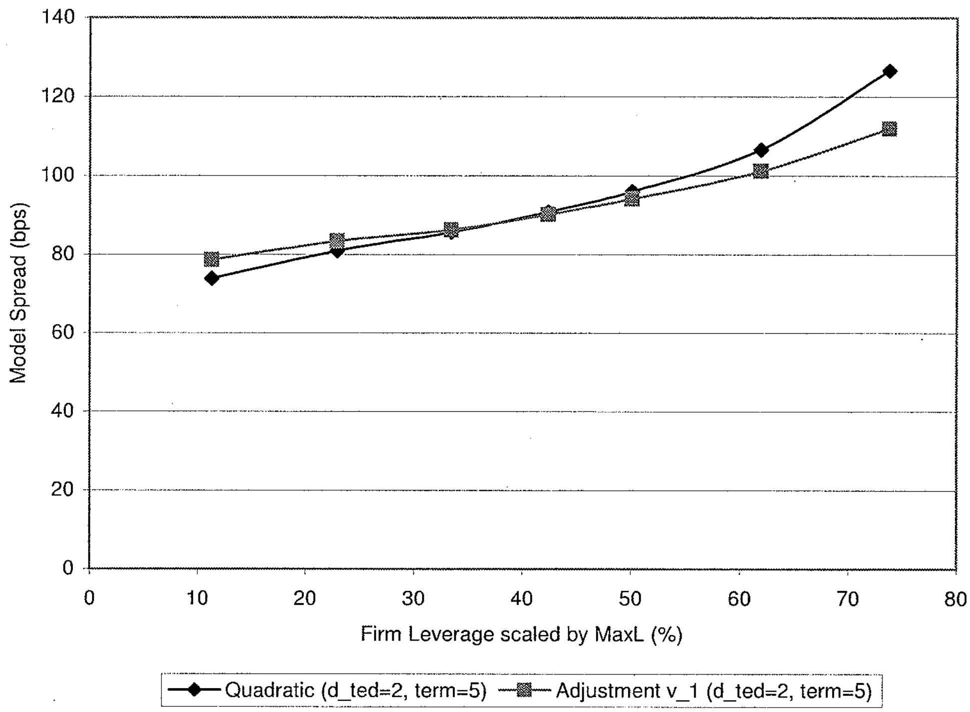

Figure 19: Adjustment v_1

A Adjustments to Model Variables in Calibration

The adjustments v_1–v_3 and w_1–w_3 regulate the impact of firm leverage, distance to expected to default, and term on leverage variance and therefore, on model spreads. The firm leverage variance is given by

$$\sigma_L^2 = c_{\text{var}} v_1(\mu_L) v_2(d_{\text{ted}}) v_3(T, d_{\text{ted}}) \mu_L (1 - \mu_L)$$

Adjustment v_1. The quadratic bound (16) is symmetric about the value $\mu_L = 1/2$ and the function

$$v_1 = (1 + 0.5(1 - \tanh(c_{\text{var1}} \mu_L))) \tanh\left(\frac{1 - \mu_L}{.15 c_{\text{var2}}}\right)$$

supplies the latitude required to account for the asymmetric relationship between the variance of firm leverage and its expected value. Figure 19 shows the impact of v_1 on model spreads as a function of firm leverage for fixed distance to expected default and term. Larger values of the parameters c_{var1} and c_{var2} reduce the sensitivity of model spreads relative to the quadratic specification of firm leverage variance.

Adjustment v_2. The default swap spread for a very creditworthy firm does not drop as rapidly as a function of distance to expected default as is predicted by a standard

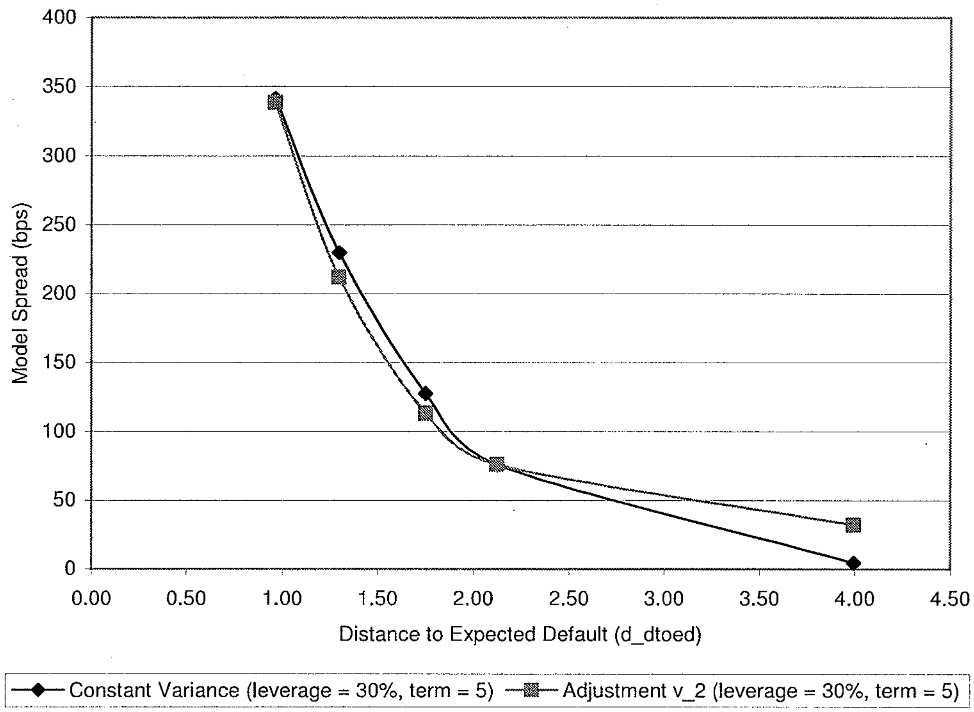

Figure 20: Adjustment v_2

structural model. This leads us to introduce the adjustment

$$v_2 = 1 + 100c_{\text{var3}} \exp(c_{\text{var4}} d_{\text{ted}})),$$

which controls the effect of distance to expected default d_{ted} on barrier variance. Figure 20 shows the impact of v_2 on model spreads. Larger values of the model parameters c_{var3} and c_{var4} increase spreads for credits with large values of d_{ted} relative to those with small values of d_{ted}.

Figure 21 shows a sample profile of firm leverage variances. Note that for a fixed value of expected firm leverage, credits with larger d_{ted} have higher firm leverage variance.

Adjustment v_3. Market default swap term structures are generally upward sloping for non-distressed firms. For high quality firms a term independent variance produces term structures that are flatter than those observed in the market. To compensate, we further adjust firm leverage variance by

$$v_3 = \left(\frac{T}{10}\right)^{\gamma}, \qquad \gamma = \frac{c_{\text{var5}}}{1 + 500\exp(-2.5d_{\text{ted}})},$$

which takes account of both term T and d_{ted}. The effect of term on firm variance needs to be tempered for credits with a smaller distance to effective default as they calibrate with a smaller variance due to adjustment v_2. Figure 22 shows the impact of v_3 for a particular combination of leverage and distance to expected default.

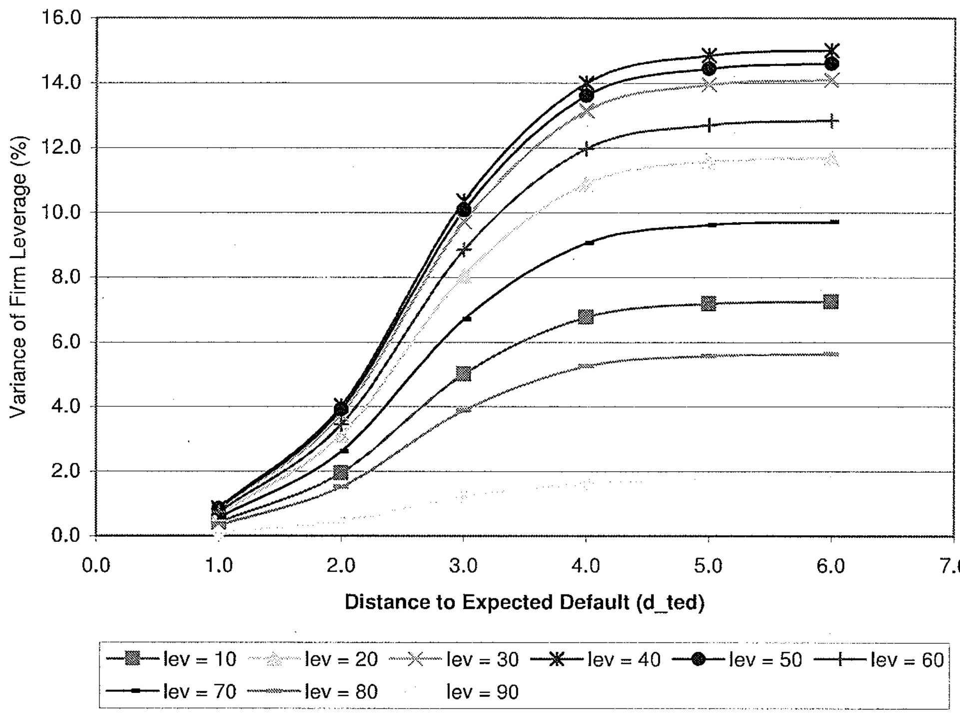

Figure 21: Profile of firm leverage variance

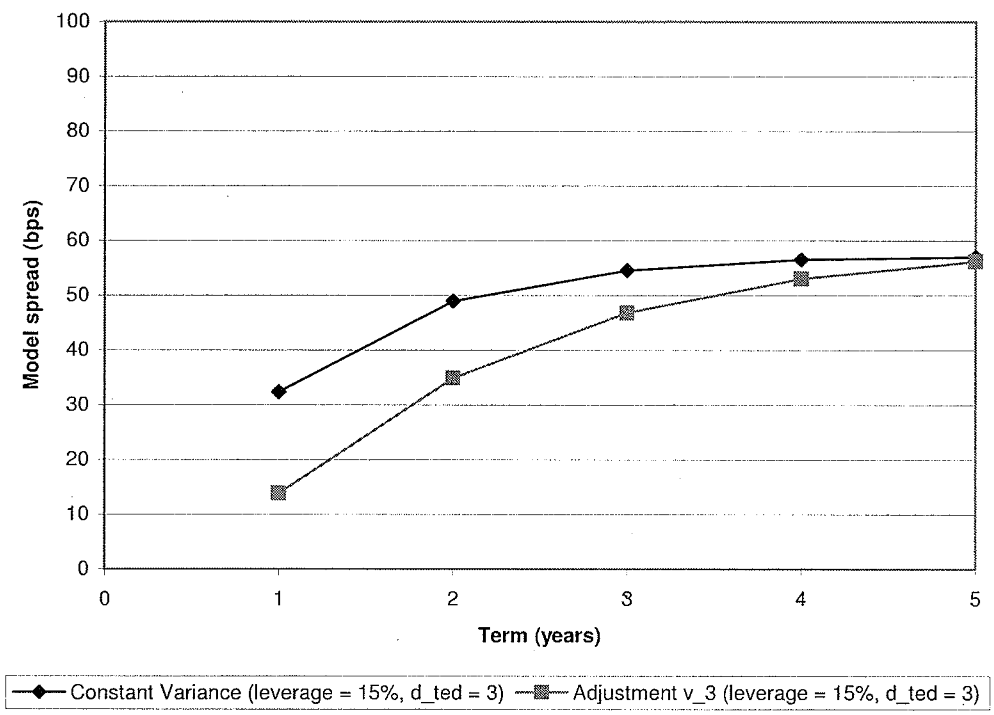

Figure 22: Adjustment v_3

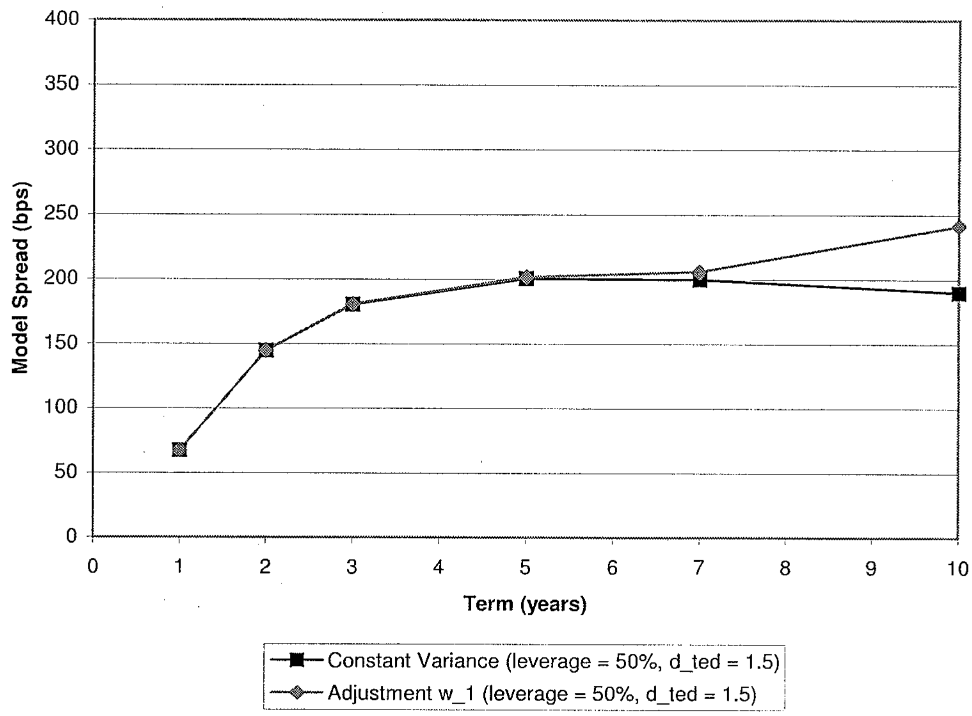

Figure 23: Adjustment w_1

Adjustment w_1. Structural models do not usually produce an upward sloping term structure for longer maturities, but rather one with a hump, with spreads flat to decreasing in the 7–10 year range. One reason for this may be that the equity volatility forecast used in the calibration has a medium term horizon while credits are evaluated with a higher volatility for long maturities.

The firm volatility σ_V generated by the inversion step is scaled by the function

$$w_1 = 1 + \frac{c_{\text{vol1}}}{(1 + 150\exp(-(T-5)))(1 + \exp(-5(\mu_L - 0.5)))},$$

which depends on term T and expected leverage μ_L. The effect of term on firm volatility is lower for credits with relatively high expected leverage μ_L. Figure 23 shows the impact of this adjustment on model spreads.

Adjustments w_2 and w_3. During periods of high volatility, there is a pronounced effect of contagion and momentum on default swap spreads. To account for this, we introduce equity return data to refine the distinction between credits within a calibration group. The result is an overall lowering of model errors.

We construct two types of risk indices that are used in the calibration process. The *contagion risk indices*, denoted I_C, are used to identify industries with recent downturns in the equity market. Precisely, for each industry, I_C is the minimum of 0 and an exponentially weighted average of 60 daily industry common factor returns. The half-life of the weighting scheme is 60 days. Figure 24 shows the history of this index for the U.S.

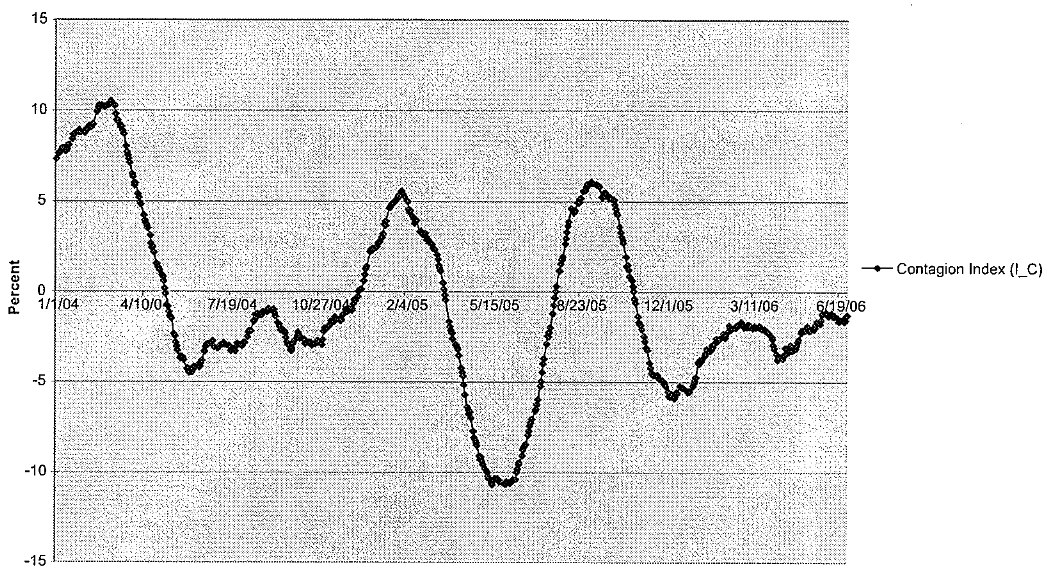

Figure 24: History for the contagion index, I_C for the U.S. Autos Industry

Autos industry which is part of the Consumer Discretionary sector. The *momentum risk indices*, denoted I_M, are used to identify credits affected by negative equity returns. They are defined analogously to the contagion risk indices, using the equity specific return in place of the industry common factor return.[6]

Figure 25 shows a scatter plot of calibration errors in a base model that has not been adjusted for contagion and momentum effects versus the appropriate contagion risk indices for the U.S. market on May 16, 2005, when markets were in turmoil due to the General Motors and Ford downgrades. Figure 25 shows a bias: errors tend to be positive for negative values of the contagion index.

The parameters c_{vol2} and c_{vol3} control the impact of the contagion and momentum risk indices on firm value volatility through the following adjustments[7]:

$$w_2 = 1 + \frac{c_{\text{vol2}}}{1 + 100\exp(66(I_C + 0.01))}$$

$$w_3 = 1 + \frac{c_{\text{vol3}}}{1 + 50\exp(10(I_M + 0.03))}.$$

In volatile periods, values of I_C and I_M become more negative. The contagion and momentum parameters c_{vol2} and c_{vol3} tend to increase. This raises the level of firm value volatility for the corresponding credits.

An example is shown in Figure 26 for the U.S. Consumer Discretionary sector for the same date. Errors for the Auto industry in this calibration group were reduced by 25 percent by including the contagion risk index in the calibration, and their distribution did not show the skewness present in the version without the adjustments.

[6]In Europe, we use the total return since specific return is not available.

[7]Based on analysis of dates around which risk indices were significant, thresholds of -1 and -3 percent were identified for the contagion and momentum risk index, respectively, only beyond which the adjustments w_2 and w_3 reduce calibration errors.

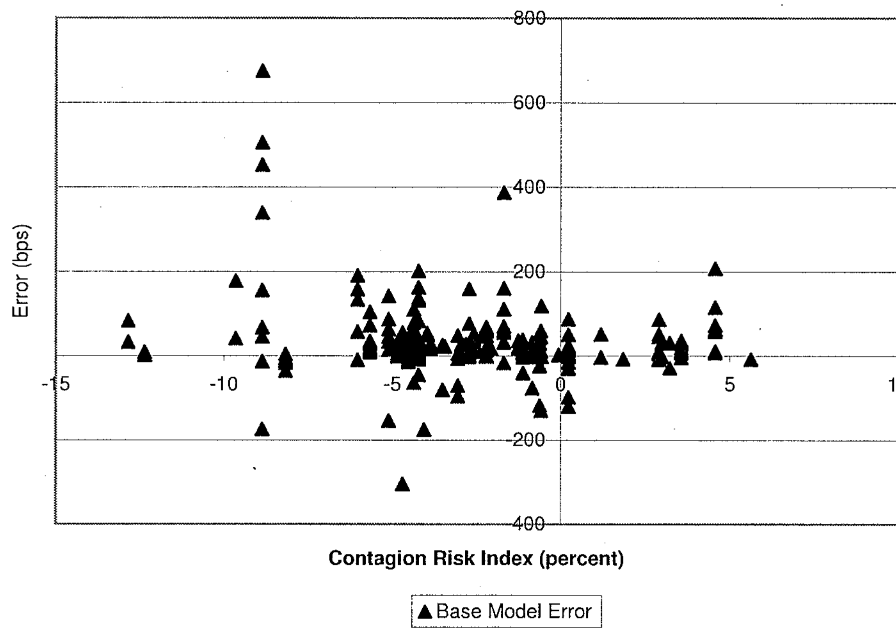

Figure 25: Base Model errors versus Contagion Risk Index Values (U.S. market, May 16, 2005).

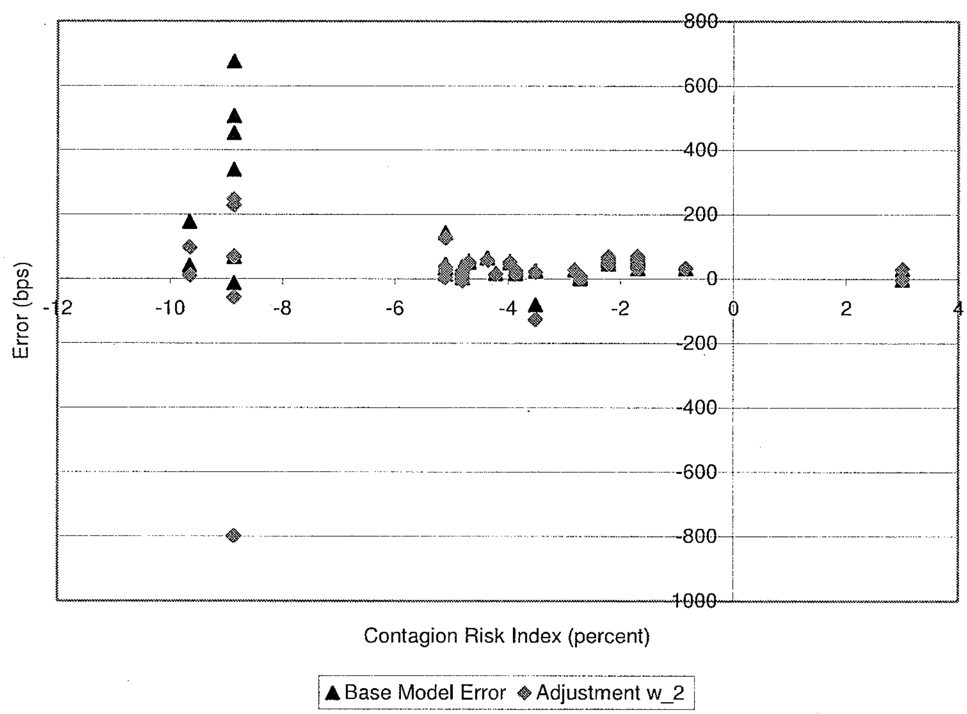

Figure 26: Adjustment w_2